that

$$f(\xi)=\int_0^\xi \exp(-\alpha\xi^2)\,d\xi.$$

The Galerkin method gives an approximation for α by solving

$$\int_0^\xi (f')^4 d\xi = 2\int_0^\infty (f')^2 f''f''\,d\xi - \int_0^\infty f'(f'')^3 d\xi$$

or

$$\int_0^\infty \exp(-4\alpha\xi^2)d\xi = 8\alpha^2\int_0^\infty \xi\exp$$

$$\times(-4\alpha\xi^2)d\xi - 8\alpha^3\int_0^\infty \xi^3\exp(-4\alpha\xi^2)d\xi$$

or

$$\sqrt{\pi}/4\sqrt{\alpha}=\alpha-\tfrac{1}{4}d$$

or

$$\alpha=(\pi/9)^{1/3}$$

Therefore, one finally has that $u/u_0=\exp[-\xi^2(\pi/9)^{1/3}]$ with ξ containing both m and c that can be selected to fit "best" the "best" data.

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SERT II Spacecraft Thruster Restart, 1974

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THE SERT II (Space Electric Rocket Test II) spacecraft was launched into a 100-km, polar, sun-synchronous orbit in February 1970 with a goal of demonstrating long-term operation of an ion thruster in space.[1] Thruster 1 was operated for 5½ months and then thruster 2 was operated for 3 months at 6.3 mlb thrust, 4200 sec specific impulse with 850 w input power. Thruster operation was terminated in each case by a high-voltage short due to an eroded web of the accelerator grid.

By 1973 the orbit had precessed such that the sun angle was oblique and only marginal power was available. To obtain more solar power the spacecraft was tipped over and spin stabilized such that the solar array was in a plane normal to the sun. A notable result was that during the first test of thruster 2 the high-voltage short was found to be cleared following the spin maneuver.

During the 1974 test period thruster 2 was restarted 19 times and run to thrust levels limited only by the available solar power. The high-voltage short remains in thruster 1, but its cathodes were started 12 times to show restart capability. The propellant feed systems, power processors, and spacecraft ancillary equipment were demonstrated to be functional after 4½ years in space.[2] In addition to thruster tests, a neutralizer cathode was operated separately to demonstrate that the potential level of a spacecraft could be controlled by the neutralizer alone.

Continued precession of the orbit will bring a continuous sun orbit in 1980 and the possibility of continuous thruster operation. Presently, shadow portions of the orbit prevent more than fractional (<1 hr) orbit periods of solar power operation.

This Note presents the highlights of the data taken during 1974. For a more complete discussion of the data and the 15-cm diameter mercury electron bombardment thruster, the reader is referred to the conference preprint[3] and earlier SERT II references.[4-6]

Thruster Operation

Thruster turn-on and operate commands were limited to real time (~20 min) periods while maintaining spacecraft contact over a ground station. Thruster 2 beam-on time varied from a few seconds to 40 min (two ground stations in sequence). Comparison data for thruster operation in 1974 with

Table 1 Performance of flight thruster 2

	30% beam		80% beam		Telemetry uncertainty (rss)
Year	1970	1974	1970	1974	
Day	2/11	9/10	2/11	9/11	
Restart number	10	198	10	200	
Main vaporizer heater	[a]1.63	1.70	1.70	1.85	±0.07 V
	[a]1.51	1.77	1.70	1.95	±0.08 A
Main cathode heater	7.9	8.7	8.3	8.7	±0.35 V
	1.54	1.57	1.54	1.57	±0.05 A
Main discharge	42.2	42.2	41.5	41.4	±0.2 V
	0.7	0.6	1.2	1.1	±0.05 A
Beam voltage	[d]3490	[d]2960	[d]3160	[d]2630	±65 V
Beam current	[d]0.088	[d]0.083	[d]0.203	[d]0.198	±0.005 A
Accelerator grid	[d]-1730	[d]-1480	[d]-1640	[d]-1330	±50 V
	1.1	0.9	1.4	1.4	±0.1 mA
Neutralizer heater	[a]6.6	8.1	[a]6.4	7.5	±0.25 V
	[a]2.0	2.3	[a]1.9	2.2	±0.05 A
Neutralizer keeper	27.8	27.8	[c]24.0	[c]27.8	±0.7 V
	[d]0.215	[d]0.175	[d]0.206	[d]0.167	±0.004 A
Spacecraft voltage	-17	-8	-17	[e]	±2 V
Neutralizer emission	0.087	0.080	0.201	0.195	±0.006 A
Main cathode keeper	20.4	20.0	13.9	13.1	±0.5 V
	[b]0.282	[b]0.272	[b]0.283	[b]0.272	[b]±0.003 A
Solar array voltage	68	59	63	52	±1.0 V

[a]Heater power lower due to higher thermal background.
[b]Estimated value.
[c]Values due to different set points.
[d]Difference in values due to different solar array voltage input to power processor.
[e]Data unavailable.

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1970 are shown in Table 1. There is good agreement within telemetry uncertainty between all steady-state parameters at both beam current levels that were compared. An abbreviated preheat was followed, but all cathodes lighted within consistent time limits and the thruster control loops regulated propellant flow and operating voltages. The thruster was turned off either by ground command or by undervoltage to the power processor. The undervoltage was caused either by drawing too much current from the solar array or by the array passing into the Earth's shadow.

Thruster System Component Status

There is little or no apparent change in any of the heaters of the SERT II thrusters. The heater resistance, as indicated by the heater voltage divided by the current, remained constant over the 5-year period from preflight qualification tests to the present. There is no apparent electrical leakage across any insulator in thruster 2.

The propellant feed system remains completely functional for each thruster. Mercury is supplied upon command from each of four vaporizers. The pressure of the nitrogen blowdown gas behind the rubber bladders of both the neutralizer propellant tanks remained constant without leaking during storage period of over a year. At present, thrusters 1 and 2 have operated their vaporizers for 3889 and 2175 hours, respectively. The design value of the propellant tanks provides for 6000 hours of flow, so thruster 2 (presently operational) has propellant remaining for nearly 4000 hours more flow.

The power processors continue to function without malfunction or noticeable degradation after 5 years in space. Each individual power supply output current and voltage agrees with its original response curve as measured in preflight qualification testing. The output voltage of the high voltage supplies and the keeper supplies varied directly with the voltage input from the solar array. All thruster set points are functional and vary slightly as predicted by original response to load or solar array input voltage. The high-voltage overload shutdown and automatic recycling continues to perform normally. All power supply telemetry outputs (also part of the power processor) remain operative.

Spacecraft and Plasma Potential Level Experiments

An objective of tests in 1974 was to see if a neutralizer operating alone could control the spacecraft potential level. An operating neutralizer should, in addition, be capable of emitting sufficient electrons if they are needed in the control of spacecraft charging. A second objective was added after thruster 2 became operational, namely, perform a neutralizer bias experiment with an operating thruster at a beam current level not previously tested, and compare the results with those taken at a higher beam current level in 1970.[7] As in Ref. 7, the potential of space is used as reference and is assumed to be zero for the following experiments and figure discussion. Neutralizer bias is obtained by a power supply placed between the neutralizer and the spacecraft body.

Neutralizer-Only Tests

Figure 1 is a plot of 1974 measurements of the SERT II spacecraft potential as a function of latitude. The quiet spacecraft floated at potential levels of near zero to −22 volts, depending on time of day (longitude, latitude, and perhaps local anomalies in space plasma). When either thruster neutralizer was turned on, however, the spacecraft potential was held between zero and −5 volts irrespective of spacecraft position. The spacecraft potential was thus driven to near zero by an unbiased neutralizer cathode without the need of an ion beam to assist in coupling the neutralizer electrons to space. In addition, each thruster neutralizer was turned on and the neutralizer bias voltage was set at −45 and −23 volts. The result at negative bias indicated a small increase in spacecraft potential in the range of +2 to +5 for −23 volt bias and +2 to +10 for —45 volt bias.

The hollow cathode therefore is a candidate cathode to perform long-term reliable spacecraft potential control. The SERT II neutralizer cathodes have operated in space for over 5 years with operating times of 3889 and 2175 hours. Recent ground tests of similar cathodes have accumulated 20,000 hours operating time on a single cathode without failure.[8]

Thruster Test with Neutralizer Bias

Thruster 2 was turned on, stabilized at 0.083 amp beam current, and the neutralizer cathode biased at normal ±25

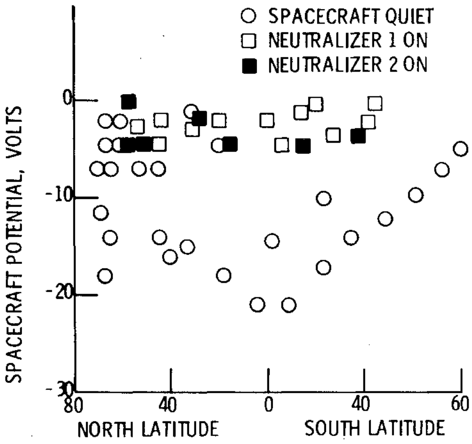

Fig. 1 SERT II spacecraft potential as a function of latitude. (Neutralizer cathode is at zero bias.)

	NEUT. BIAS.	SPACE-CRAFT	ΔV_{BS} v	ΔV_{BN} v	LATITUDE	NEUT. CATHODE EMISSION a	NEUT. CATHODE POTENTIAL v
○	+44.5	-43±2	86	41	28S	0.083	+2
□	+22.1	-28±1	59	37	34S	0.083	-6
△	0	-8±1	38	38	20S	0.083	-8
◇	-22.8	0±1	21	44	38S	0.130	-23
◺	-45.2	0±1	0	59	42S	0.195	-45

ΔV_{BS} - DIFFERENCE, BEAM CENTER-TO-SPACECRAFT
ΔV_{BN} - DIFFERENCE, BEAM CENTER-TO-NEUTRALIZER

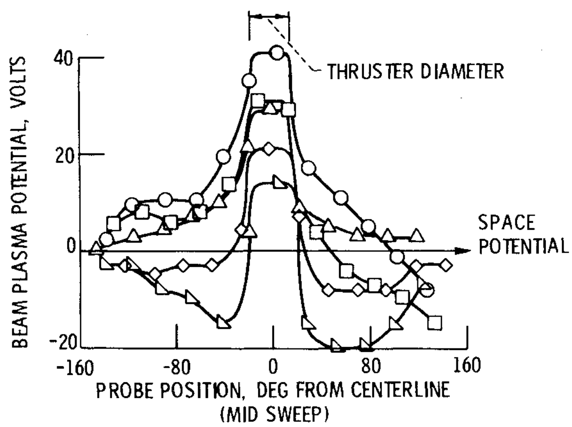

Fig. 2 Beam plasma potential profiles at various neutralizer bias potentials. Spacecraft, neutralizer cathode, and beam plasma voltages are relative to space plasma voltage which is assumed zero.

and ± 50 volts. A hot-wire probe was swept through the beam at each bias and zero bias to measure the beam plasma potential and the results are shown in Fig. 2. The beam probe was only in the ion beam approximately $\pm 20°$ about the thruster centerline which coincided with the midpoint of the probe sweep. The balance of the sweep measured the plasma potential in the fringe or wing area of the beam plasma.

The data shown on Fig. 2 agreed with the results of Ref. 6. The SERT II spacecraft tended to float at 0 to 20 volts below space potential with no thruster or neutralizer on. With a thruster or neutralizer on the spacecraft could be maintained near zero potential or biased negatively. Positive bias of the spacecraft was ineffective because the neutralizer emission current was preferentially drawn to the spacecraft rather than space plasma.

Summary

The SERT II spacecraft, designed for 1 year life, remains functional after 5 years in space. Opportunity exists therefore to check the long-term operational status of the on-board ion thruster components, power processors, and other spacecraft ancillary equipment. During the 1974 test opportunity reported in this Note, a notable result was that the high-voltage short was clear on thruster 2 and that normal thruster operation was restored within the limits of available solar power. The cathodes and propellant supply system continued to function normally on thruster 1. Both power processors continued to function without fault after 5 years in space and 3889 and 2175 operating hours, respectively.

In addition to the thruster tests, a neutralizer cathode was operated separately to demonstrate that the electric potential level of a spacecraft could be controlled by the neutralizer. Orbital mechanics predict a continuous sun-lighted orbit in late 1980. If spacecraft reorientation maneuvers are performed, it could be possible to operate thruster 2 continuously in a 1981 test opportunity with the propellant remaining in the thruster reservoirs.

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Role of Thermal Contact Resistance in Pyrotechnic Ignition

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SPACECRAFT have been relying on small charges of pyrotechnic inside squibs to do useful work in space. Such work includes separating spacecraft from their launch vehicle, unfolding solar panels, and releasing communications antennas. A squib usually consists of a threaded metal body that holds a small amount of pyrotechnic in one end and has electrical leads sealed in glass running through the middle to the other end. A small diameter wire, called a bridgewire, is welded across the two electrical leads, and is imbedded in the pyrotechnic. An electrical current supplied from a charged capacitor, or some other source, heats up the bridgewire which in turn heats up the pyrotechnic until it ignites. This Note will examine how a thermal contact resistant can influence the time to ignition and how it can influence the instantaneous temperature distributions within the bridgewire-pyrotechnic system.

It has been shown by Sernas[1] that axial conduction in the bridgewire is negligible for short heating times that were of interest in this study. When it is assumed that bridgewire is completely surrounded with pyrotechnic, the heat transfer can be treated as a one-dimensional radial heat conduction problem in the pyrotechnic and bridgewire domains. A detailed discussion of this assumption can be found in Refs. 1 and 2.

The governing equation in the bridgewire was then

$$\rho_1 c_1 \frac{\partial T}{\partial t} = H + k_1 \left[\frac{1}{r} \frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial r^2} \right] \tag{1}$$

where ρ is the density, c is the specific heat, k is the thermal conductivity, T is the temperature, and t is the radial coordinate. The subscript 1 is used to denote the thermal properties of the bridgewire. The quantity H is the rate of volumetric heating in the bridgewire due to a constant electric current being applied to the bridgewire at time zero.

In the pyrotechnic the governing equation was taken to be

$$\rho_2 c_2 \frac{\partial T}{\partial t} = k_2 \left(\frac{1}{r} \frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial r^2} \right) \tag{2}$$

where the subscript 2 denotes the properties of the pyrotechnic. A term representing the self-heating of the pyrotechnic due to an exothermic chemical reaction has been left out to simplify the model.

The boundary condition used at the interface between the bridgewire and the pyrotechnic was

$$k_1 \left(\frac{\partial T}{\partial r} \right)_1 = h(T_1 - T_2) = k_2 \left(\frac{\partial T}{\partial r} \right)_2$$

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